STRAYER-UPTON
PRACTICAL ARITHMETICS
THIRD BOOK

BY

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PREFACE

THIS series of books aims to give the child the ability to compute easily and accurately and to interpret and solve the quantitative situations which he will meet in everyday life. In the achievement of this aim, these books incorporate the most valuable findings of modern experimentation in the teachings of arithmetic, including the results of important researches conducted by the authors. These books present only those methods and materials which have been thoroughly tested in the classroom by many experienced teachers.

It is an established principle of teaching that nothing stimulates a pupil's interest so much as the satisfaction he gets from his ability to do things successfully. If he finds himself steadily perfecting new skills in arithmetic, he will look forward to his arithmetic period and will approach each new topic with intelligent interest. In accordance with this principle, these texts require the pupil to take only one new step at a time and supply him with enough exercises to assure mastery of that step before proceeding to the next one. This careful development of the work applies not only to the fundamental operations with whole numbers, fractions, and decimals, but also to percentage, graphs, and the business applications of arithmetic. Thus the constant stimulus of success is made possible for the pupil.

In this book the following features are worthy of note:

1. This book meets the requirements of the leading curricula in mathematics for the seventh and eighth school years. It also follows the recommendations of the National Committee on Mathematical Requirements and the Year-books of the National Council of Teachers of Mathematics.

2. Motivation is the keynote of successful work in mathematics. When new work is being presented, the pupils should know why this work is important and how it is used in life. This fundamental principle of teaching is applied throughout this book by presenting every new topic in connection with an interesting motivating problem that treats of some situation with which the pupil is familiar.

3. The applications of arithmetic as found in business and industry, as well as those encountered in personal life, are presented in such a way as to make them real and interesting to boys and girls of the upper grades. Special attention has been given to such topics as banking, modern business practice, accounts, budgets, thrift, savings, investments, insurance, installment buying, purchasing a home, and taxation.

4. This book contains many interesting projects and units of work, including much valuable material relating to transportation, communication, and American industries.

5. Form study and geometric constructions are presented through a series of attractive exercises in actual drawing, thus utilizing the pupil's natural interest in constructive work. In addition, there is a modern treatment of mensuration in which the formulas for finding areas and volumes are developed experimentally by classroom exercises, thus giving the pupil a confidence in the rules that he follows. Work on indirect measurement by means of similar triangles and on the Pythagorean theorem is also given. There is a wealth of problem material based on situations with which the pupil is familiar.

6. The formula and the simple equation, two of the most effective mathematical instruments, are fully developed as important aids to arithmetic. The formula is applied throughout the work in mensuration. The equation is employed in ratio and proportion and in the indirect cases in percentage.

7. Both the making and the interpretation of graphs, such as those which appear so frequently in newspapers and magazines, have been developed and graded with unusual care.

8. The checking of work is emphasized throughout the book.

9. A series of 94 Improvement Tests, covering whole numbers, fractions, decimals, and percentage, is included. These tests provide the most efficient and interesting means yet devised for keeping alive computational skills already learned while the pupil is studying the important applications of mathematics to everyday life. These tests have the further advantage that the total time required to give and score them is less than 30 minutes a week, thus leaving ample time to be devoted to the new work of the grade. For further information concerning these tests, see pages 13–22 and 529 of this book.

10. This book provides an exceptionally full and generous program of diagnostic and remedial work. Frequent diagnostic tests are given throughout the book, with keyed references to remedial exercises.

11. The carefully planned instruction in problem solving which was given in the first two books of this series is continued in this book. There are frequent reviews of the technical expressions of arithmetic and of the more important types of two-step and three-step problems. There are also many pages containing problems of various kinds, each of which requires the pupil to think carefully before planning the solution of the problem. Besides this material, the pupil is introduced to many new problems that treat of the business, social, and economic life of to-day. Special attention is called to the fact that this new problem material, whether it relates to banking, investment, taxation, or any other topic, is truly representative of conditions as they exist to-day. Artificial problems or obsolete business practices are not to be found in this text.

12. A series of tests on problem solving is also provided throughout the book. These tests cover types of problem situations with which every pupil should be familiar. An important feature of these tests is that they not only measure the pupil's mastery of types of thinking frequently employed in problem solving, but they also check his knowledge of important business and social applications of arithmetic. These tests are arranged in groups known as Groups A, B, and C. Each group consists of three tests; thus, Group A consists of Tests A 1, A 2, and A 3. The problems on Test A 1 cover the same variety and types of problem situations and the same range of difficulty as those found on Tests A 2 and A 3. A pupil, therefore, should do better on the second and third tests of any group than on the first test. Thus the pupil has the satisfaction of seeing himself grow in problem-solving ability. The problems on the tests of Groups B and C cover, in similar manner, other sets of type problems.

13. Full provision has been made for pupils of varying levels of ability. For those of average and below-average ability a large number of well-graded exercises is provided. For pupils of superior ability more difficult exercises, marked with a star (★), are furnished. The diagnostic tests indicate the needs of each pupil and give references to suitable remedial work.

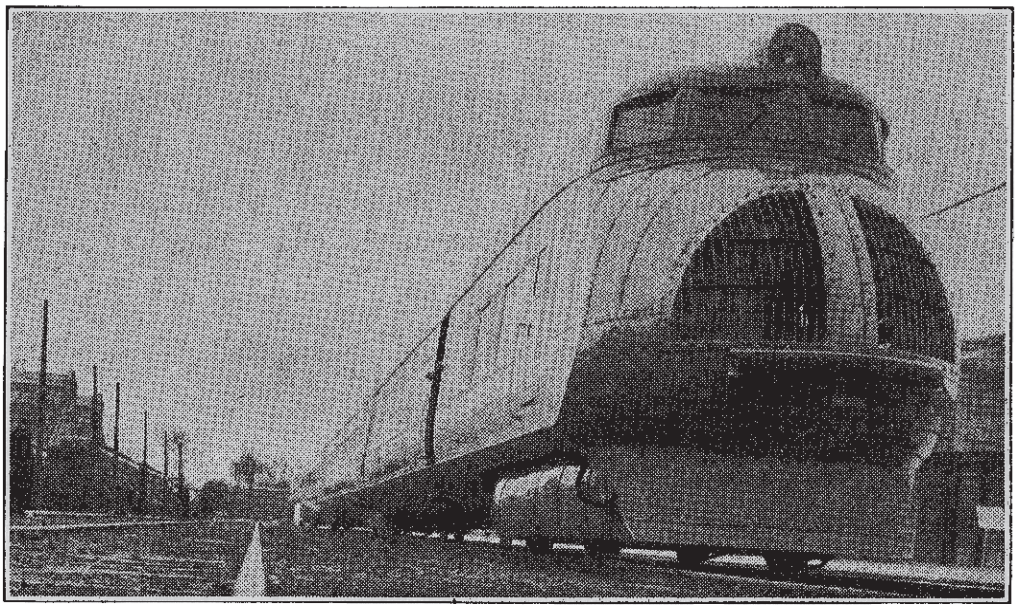
14. The liberal amount of material offered in this book permits a selection of topics in accordance with local needs. Changes may also be made in the order of presenting a number of the topics without disturbing the unity of the work.

George Drayton Strayer
Clifford Brewster Upton

CONTENTS

CHAPTER I

TRANSPORTATION AND COMMUNICATION



THE RAILWAY TRAIN OF TO-MORROW

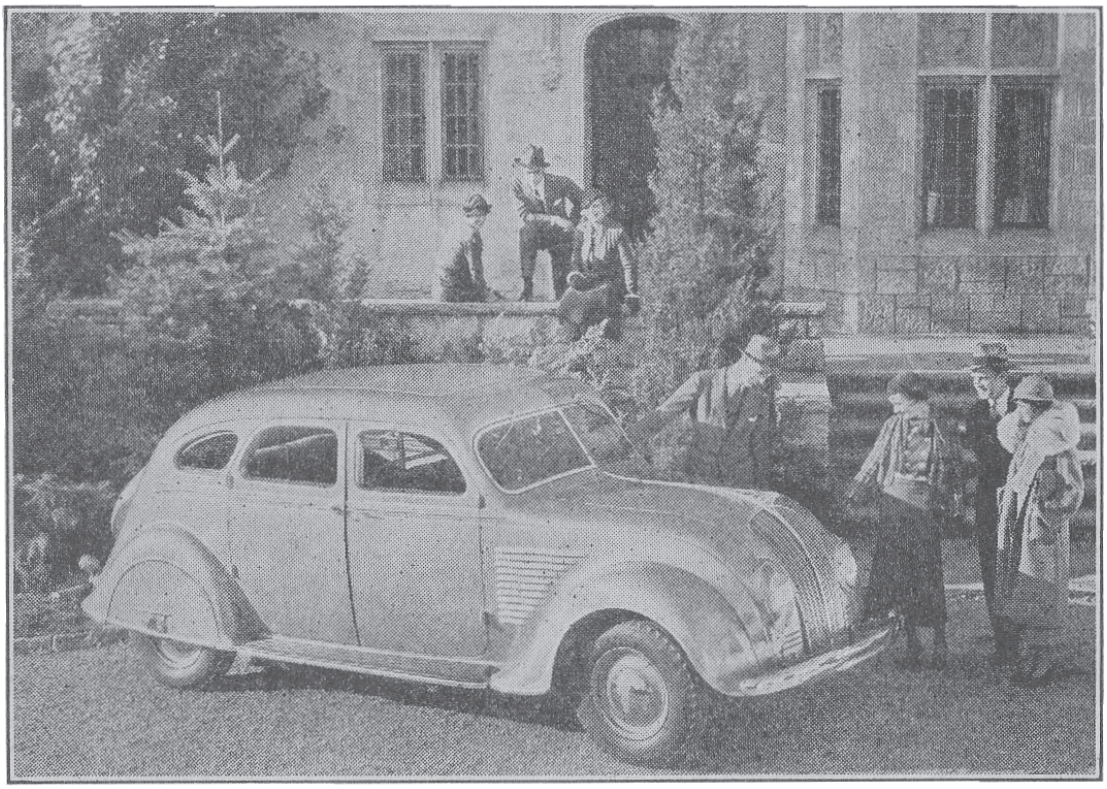
This new type of electric stream-line train, built of aluminum alloy, can run at an average speed of 90 mi. an hour.

1. The fastest steam express train from Chicago to the Pacific Coast now takes 56 hr. to make the trip of 2228 mi. Find its average speed per hour.

2. At 90 mi. an hour, how many hours will it take the new stream-line train to make this trip of 2228 mi.?

3. When this new train was first described, the newspapers stated that its speed was 90 mi. an hour and that it would make the trip of 2228 mi. in 32 hr. Were the newspapers right? If the train took 32 hr., at what speed would it be running?

AUTOMOBILES AND BUSES



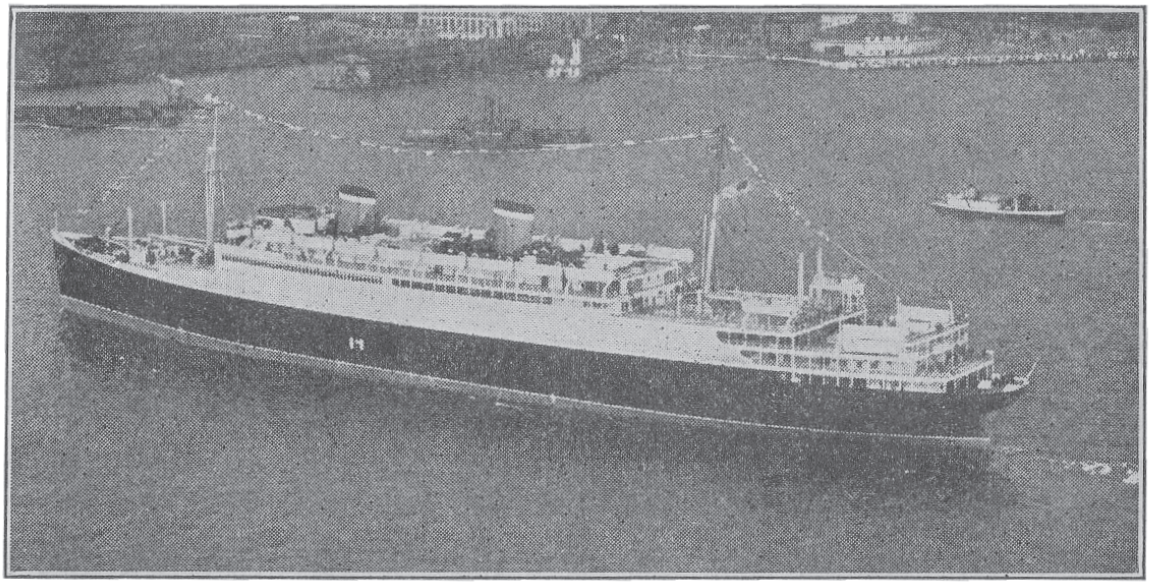
1. In the year 1930 about 23,000,000 passenger automobiles were registered in the United States. If the total population of the United States that year was 122,000,000, was there an average of 1 automobile to about every 4 people or 1 to every 5 people or 1 to every 6 people?

2. In the year 1920 there were 8,000,000 passenger cars registered in the United States. The same year the population of the United States was 105,000,000. This was an average of 1 automobile to about how many people?

3. In the year 1900 there were 8000 passenger cars in this country and the population was 76,000,000. This was an average of 1 automobile to how many people?

4. In one year the motor buses of this country carried 2,274,300,000 passengers, while the railroads carried 478,800,000 passengers. The motor buses carried about how many times as many passengers as the railroads?

OCEAN STEAMSHIPS



1. One of the largest and fastest ocean liners of the present time is 880 ft. long. How many ships of the same length, placed end to end, would extend a mile?

2. On a record trip a steamship crossed the Atlantic, a distance of 3248 nautical miles, in 4 da. 15 hr. Find the average speed per hour, carrying the result to two decimal places. A nautical mile equals 1.15 land-miles. Also find the speed of this steamship in land miles per hour.

3. To make a fast run takes an unusual amount of coal. On a record transatlantic trip of 4 da. 17 hr. 48 min. another steamship used 1 T. of coal every $1\frac{1}{2}$ min. for the entire trip. How many tons of coal were burned on the trip?

4. From this table tell by how much our total exports, carried by ships in one year, exceeded our total imports.

Exports		Imports	
To Europe	$1,186,884,000	From Europe	$640,096,000
To Asia	386,354,000	From Asia	574,301,000
To S. America	158,691,000	From S. America	307,191,000
To Africa	59,959,000	From Africa	32,884,000

CARRYING PASSENGERS BY AIRPLANE

1. American-operated transport planes are now carrying 551,000 passengers a year over established routes. This is 95 times as many passengers as they carried 7 years ago. How many passengers did they carry 7 years ago?

2. At present express passenger airplanes cover the 2609 mi. between New York and Los Angeles in 16 hr. This is at the rate of how many miles an hour?

3. The latest air liners have a cruising speed of 3 mi. a minute. At this rate how long will it take one of these airplanes to fly from Pittsburgh to Indianapolis, a distance of 342 mi.?

4. The average cost to a passenger for a trip on an air line in the United States is 6.1¢ a mile for a one-way fare. At this rate, find the approximate cost of each of these trips:

Omaha to Seattle	1785 mi.	Kansas City to Dallas	538 mi.
Boston to Chicago	910 mi.	Chicago to Atlanta	640 mi.

This table shows the growth in the number of passengers carried by airplanes in the United States and in Europe:

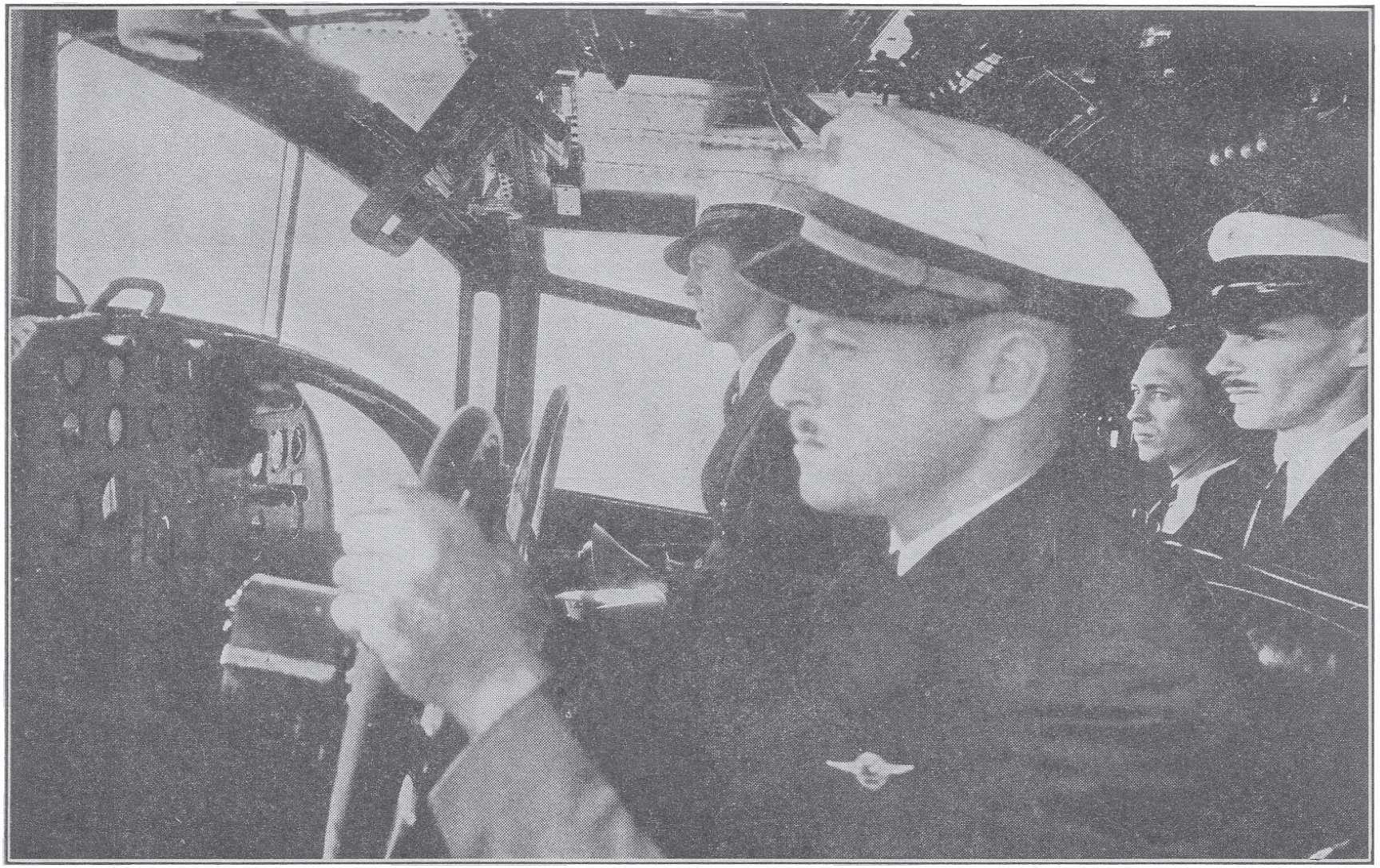
Year	United States	Europe
1926	5,782 passengers	150,095 passengers
1928	52,934 passengers	222,036 passengers
1930	385,910 passengers	251,332 passengers
1932	504,575 passengers	318,743 passengers

5. In 1926, European airplanes carried about how many times as many passengers as did American airplanes?

6. In 1928, American airplanes carried about how many times as many passengers as they carried in 1926?

7. In what years did American airplanes surpass European airplanes in the number of passengers carried, and by how many?

8. In 1932, American airplanes carried about how many times as many passengers as did European airplanes?



(5) Pilot, Co-Pilot, Mechanician, and Radio Operator of a 44-Passenger American Airplane on Its Way from Florida to South America

AIR MAIL AND EXPRESS

1. In one year about 750,000 letters were sent by air mail every day. If the average weight of each letter was .4 oz., how many pounds of letters were sent by air mail that year?

2. The same year letters and packages sent by air mail had a combined weight of 7,665,000 lb. How many pounds of packages were sent by air mail that year? See ex. 1.

3. In 1932 the cost per mile of carrying air mail was $54\frac{1}{3}¢$ in the United States and $88\frac{7}{10}¢$ in Europe. The European cost per mile was how many cents more than the American cost?

4. In 1932 the air mail was carried 32,202,170 mi. at an average cost to the government of 62¢ a mile. If the sale of air-mail stamps amounted to $9,899,000 that year, how much did the carrying of the air mail actually cost the government?

Airplanes also carry many kinds of express, such as fruits and vegetables, medicines, and rush orders for merchandise. Since 1926 the weight of air express has grown rapidly, as shown below:

1926 . . .	6,467 lb.	1930 . . .	286,798 lb.
1927 . . .	12,495 lb.	1931 . . .	885,164 lb.
1928 . . .	35,376 lb.	1932 . . .	1,324,428 lb.
1929 . . .	197,538 lb.	1933 . . .	1,660,000 lb.

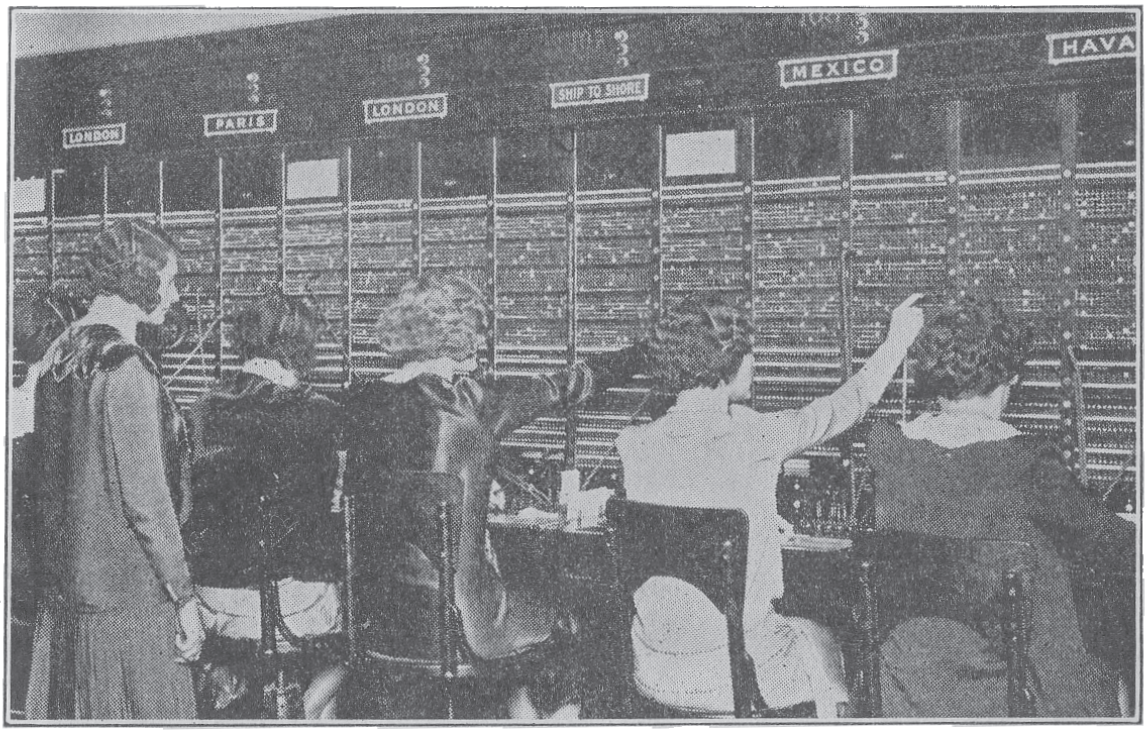
5. The weight of air express carried during the four years from 1930 to 1933 inclusive was how many times the weight of that carried the preceding four years?

6. The weight of air express carried in 1929 was about how many times the weight of that carried in 1928?

7. The air express carried in 1933 was about how many times as heavy as that carried in 1926?

8. American-operated air lines fly a total of 164,500 mi. a day, with $\frac{2}{5}$ of this mileage flown at night. How many miles each year represent daytime travel?

THE USE OF THE TELEPHONE



SWITCHBOARD FOR OVERSEAS TELEPHONE SERVICE

1. There are now 17,857,000 telephones in the United States. If there are 125,000,000 people living in this country, this means 1 telephone for about how many persons?

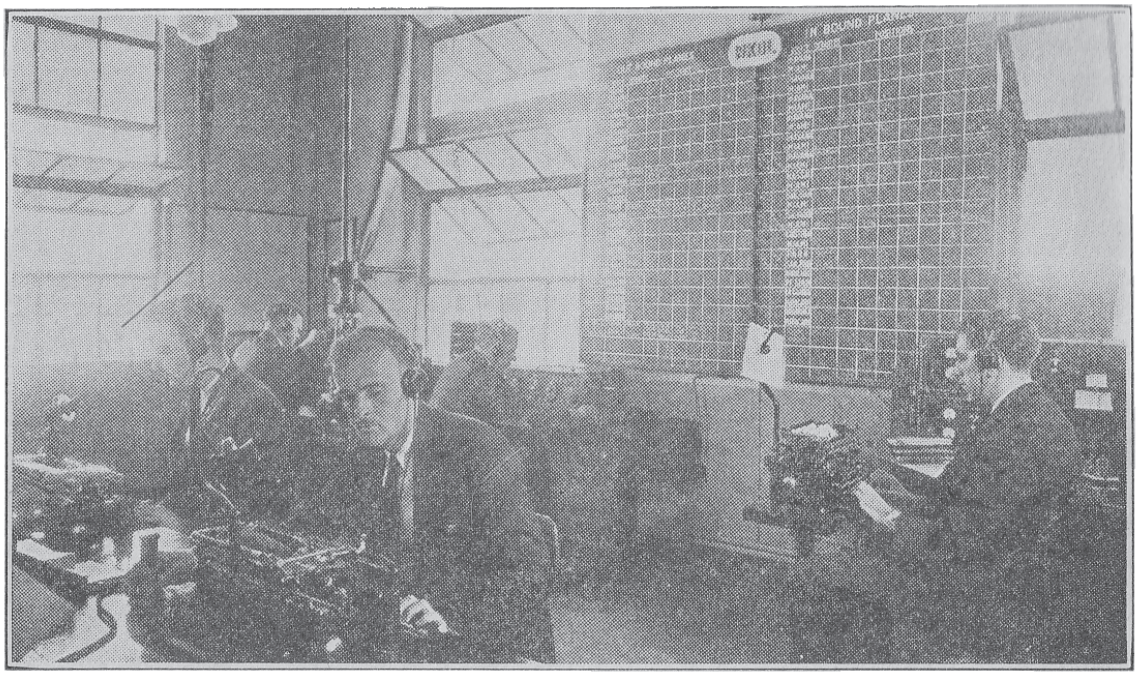
2. In 1895 there were only 339,500 telephones in the United States. About how many times as many telephones are in use now as there were in 1895?

3. There are about 6,289,000 farms in the United States and about 2,139,000 of them have telephones. That is 1 telephone for about how many farms?

4. On the average, nearly 900 telephone conversations take place in the United States every second. At that rate, how many conversations are there per year? Let 365 da. = 1 yr.

5. Ten years ago it took 12 min. to make a long-distance telephone connection. To-day it takes 2 min. If a man had 4 such calls to make, each 3 min. long, how many times as long would it have taken him ten years ago as it would to-day?

THE TELEPHONE



A Ground Station Sending Weather Reports and Other Messages to Airplane Pilots by Radio Telephone

1. The United States has 17,857,000 of the 33,400,000 telephones in the world. About what fractional part of the world's telephones are in the United States?

2. Overseas radio telephone service now makes it possible for our telephones to connect with $\frac{9}{10}$ of the world's telephones. How many of the world's telephones can we connect with?

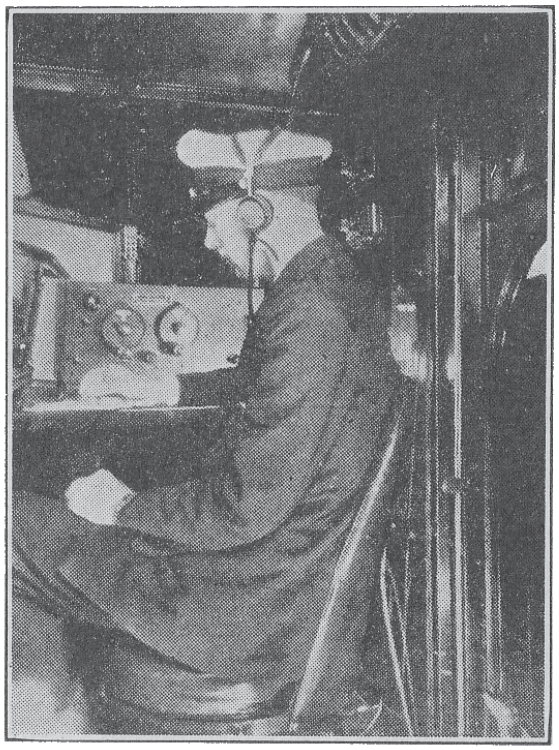
3. In this country there are 86,100,000 mi. of telephone wire. If the circumference of the earth is about 25,000 mi., how many times would this wire reach around the earth?

4. The average distance from the earth to the moon is 239,000 mi. About how many times would this telephone wire reach from the earth to the moon and back again?

5. There are 16,000,000 telephone poles in the United States. Suppose that the average diameter of these poles is 9 in. and that they are placed side by side in a solid fence. About how many miles will this fence extend?

TELEGRAPH AND TELEPHONE

1. The first telegraph message was sent in 1844 by Morse. The first telephone conversation was held in 1876 by Bell and his assistant. The first wireless telegraph message was sent across the ocean in 1901 by Marconi and the first transatlantic radio telephone service was inaugurated in 1927. How many years old is each of these four inventions?



AN AIRPLANE RADIO OPERATOR TALKING WITH AN AIRPORT BY RADIO TELEPHONE

2. In one year in this country an average of 488,500,000 words were telegraphed each month. If 195,400,000 telegrams in all were sent that year, find the average number of words per telegram.

3. Telephone conversations numbered 28,382,400,000 the same year that 195,400,000 telegrams were sent. About how many times as frequent as telegrams were telephone conversations in this country that year?

4. Find the cost of a 9-minute overseas radio telephone conversation between New York and Paris, France, if the rate for such conversations is \$31.50 for the first 3 min. and \$10.50 for each additional minute.

5. In 1933 there were 20,187 overseas radio telephone messages, of which 2361 were ship-to-shore messages, 13,097 were transatlantic messages, 3474 were Central and South American messages, and the rest were transpacific messages. How many transpacific radio telephone messages were sent?

COMMUNICATION BY RADIO

1. There were 612 radio broadcasting stations in the United States in a recent year, 52 being located in New York, 43 in California, 40 in Pennsylvania, 36 in Illinois, 34 in Texas, 26 in Iowa, 20 in Ohio, 19 each in Michigan and Missouri, 16 each in Indiana and Wisconsin, and 15 in New Jersey. How many broadcasting stations in all were located in other states than those named above?

2. These radio broadcasting stations were so located geographically that .94 of the 125,000,000 people of the United States could have heard daytime programs if they had listened over radio sets. How many persons could have listened to the daytime radio programs?

3. In 1930 there were 29,904,663 families in the United States, 12,078,345 of which had radio sets. How many families that year did not have radio sets?

4. In 1933 there were 16,809,562 families having radio sets. If a family averages 4 persons, how many more persons could listen to the family radio sets in 1933 than 1930?

5. If the population of the United States in 1933 was 125,000,000, what part of our total population lived in homes having radio sets?

6. In 1933 the states having the greatest number of families with radio sets were: California, 1,067,705; Illinois, 1,359,995; New Jersey, 818,865; New York, 2,500,723; Ohio, 1,136,142; and Pennsylvania, 1,563,127. About what fractional part of the 16,809,562 families having radio sets lived in these 6 states?

7. Radio listeners paid in one year $69,550,000 for radio tubes, $6,000,000 for parts for radio sets, $17,000,000 for batteries, $8,580,000 for accessories, besides $212,040,000 for radio sets. What was the total amount of money that was paid by radio listeners in the United States that year?

CHAPTER II

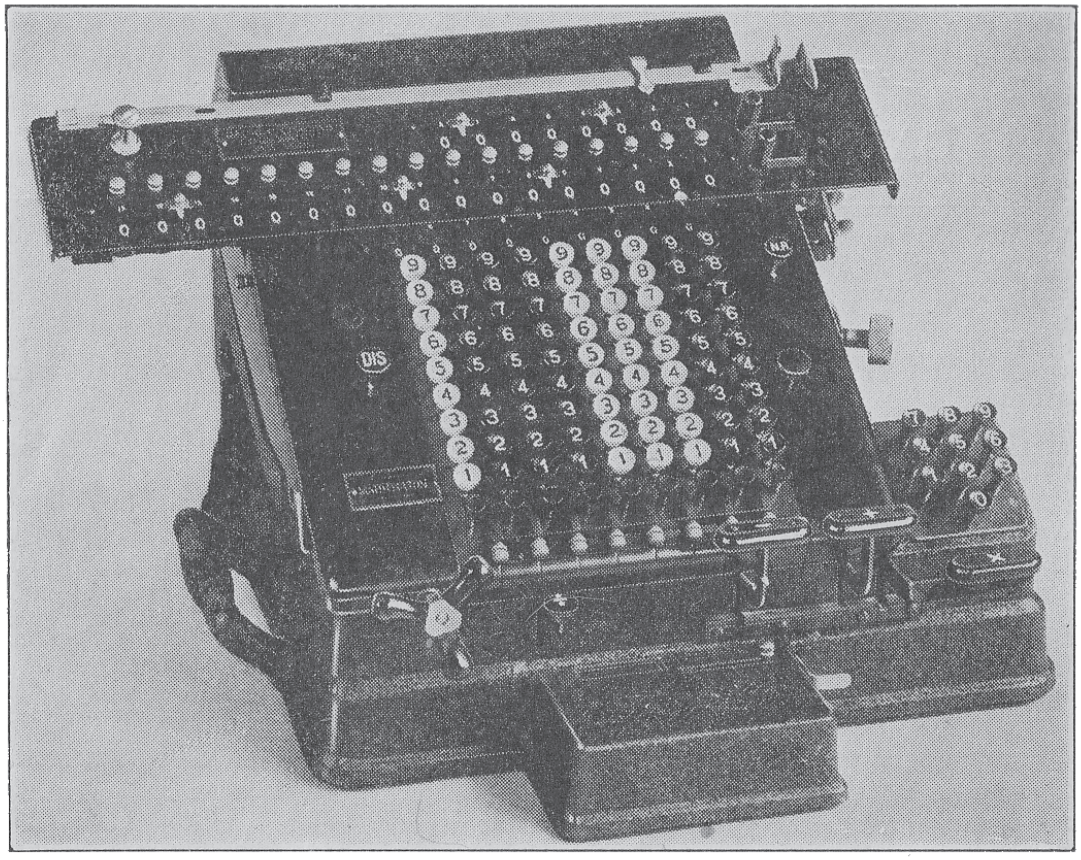
GAINING SKILL IN COMPUTATION

USING AN ADDING MACHINE

The young woman shown above is using a modern adding machine. The sheets of paper on the table contain long columns of numbers which she is adding. As she reads each number, she quickly presses the proper keys on the machine, just as one presses the keys on a typewriter. The moment she finishes the last stroke, the correct sum of all the numbers appears on dials in front of the machine.

An experienced operator of this machine can add each example on page 24 of this book in 8 seconds. How long will it take her to add all the examples on that page? This machine may be used also for subtraction, multiplication, and division.

AN ELECTRIC CALCULATING MACHINE



This wonderful machine is more than human. If you wish to multiply 6784 by 4573, you press the keys for 6784 on the large keyboard and the keys for 4573 on the small keyboard at the right side of the machine. You then touch the multiplication lever under the small keyboard and in 6 seconds the correct product appears in the dials at the top of the machine, all the work being done by electricity. The machine will also divide one large number by another in about 10 seconds.

Calculating machines are frequently used to-day in big stores, factories, and banks. There is a large amount of computing in business and in everyday life, however, that has to be done by pencil. Furthermore, this pencil computing must be done quickly and accurately if it is to be of much service. It is important, therefore, for you to improve your skill in computing as quickly as possible.

HOW TO BECOME A GOOD COMPUTER

To play a violin well or to become a good athlete requires a great amount of practice. If you do not have the grit to spend the necessary time in practice, you will not make much headway. Practice, however, must be carefully planned. If you practice too little, you will not gain much; if you practice too long, you may become fatigued. A happy medium is necessary, but you must remember that it takes time to make progress.

Suppose that John is trying to win a place on the school basket-ball team. Every day he practices shooting the ball into the basket and records carefully the number of baskets he makes. Yesterday he threw the ball in on 13 out of 30 trials; to-day he tries to get at least 14 out of 30. Only by constantly improving his record can he gain real skill.

In the same way you can gain skill in computing by watching your record and always trying to improve it. Begin by taking Improvement Test 1 A, on page 15. The teacher will time you, telling you when to start and when to stop. She will then read the correct answers and you should make a record of your score, using the table on the inside of the back cover of this book to find your score. When you take the test again, in a day or two, try hard to raise your mark.

The daily record of your scores, which you will keep in a notebook, will show whether your skill is increasing. You should also compare your score with the daily median score of the entire class, which will be kept on the board. *Try to get every example on the test right.* If you cannot do this, your score will be considered satisfactory only if it is equal to, or above, the median score of the class.

The median score of a class, which is much like the average score, is explained on page 19. The median score is used instead of the average score because it is much easier to compute.

HOW TO TAKE AN IMPROVEMENT TEST

In taking an Improvement Test in addition or subtraction, like Test 1 A or 1 C on the next page, do not copy the examples. Instead, lay a sheet of paper on the book with its edge directly under the top row of examples. Write only the answers on the paper, as shown in the picture.

When you finish the first row of examples in a test, fold the answers under and place the folded edge of the paper directly under the second row of examples. Write the answers to these examples on the paper along the folded edge. By writing only the answers, you save time and also avoid the possibility of making mistakes in copying.

Work on each test only the number of minutes permitted for that test and *stop all work immediately at the signal for stopping*, even if you are just finishing an example. You cannot compare your scores from day to day on the same test, to show whether you are improving, unless you work upon that test exactly the same number of minutes each time you take it.

The teacher will usually have you do the same test on several successive days, until the class "hits the roof" by making a median score of 10. Then she will start a new test and continue with it until the class again reaches a median score of 10. If your score is not up to the class median when the class leaves a test, practice the test by yourself. Then, when the class tries that test again after a few weeks, you should be able to equal the class median.

IMPROVEMENT TEST No. 1

Test 1 A — Addition (Time 4 min.)

1.
37120
93837
84778
96363

2.
47457
58486
49305
54875

3.
98598
65871
26389
15108

4.
98724
55965
53794
24036

5.
46975
41698
34062
12528

6.
82149
97741
47332
18385

7.
64238
81874
77676
44529

8.
39146
92779
58989
17336

Test 1 B — Addition (Time 4 min.)

1.
8133
4145
5676
6909
4739
7243
7846
3805

2.
1956
1731
5694
2859
9087
8750
3628
7263

3.
5436
2393
8722
1100
6687
8917
6478
2233

4.
5039
9564
4456
5009
8785
6984
8260
2136

5.
6455
4099
9853
7878
9760
7502
3759
1446

Test 1 C — Subtraction (Time 4 min.)

1.
16441
12053

2.
37324
28842

3.
81605
67945

4.
51000
33265

5.
38311
22361

6.
96028
38122

7.
96530
42792

8.
74254
64634

9.
96442
89764

10.
69220
33606

11.
30409
21831

12.
73879
71780

13.
62891
43512

14.
73006
15595

15.
80463
19603

16.
74463
38014

FINDING SCORES IN IMPROVEMENT TESTS

This table will help you to find your score on a test.

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	0	3	7	10													
4	0	3	5	8	10												
5	0	2	4	6	8	10											
6	0	2	3	5	7	8	10										
A → 8	0	1	3	4	5	6	8	9	10								
9	0	1	2	3	4	6	7	8	9	10							
10	0	1	2	3	4	5	6	7	8	9	10						
12	0	1	2	3	3	4	5	6	7	8	8	9	10				
15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	
16	0	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10

Suppose that you take Test 1 A and get 5 examples right. Since there are 8 examples in the test, first find in the table the *row* beginning with 8 (as shown by Arrow A). Then look *along that row* until you reach the *column* with 5 at the top (shown by Arrow B) and you find the number 6 (shown by Arrow C), which means that your score on the test is 6.

In finding your score, *count as correct only the examples that are entirely right.* An unfinished problem does not count.

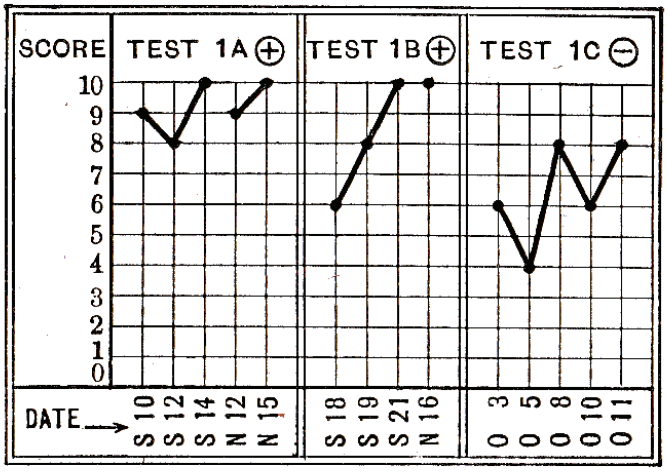
USING THE SCORING TABLE

1. In Test 1 A, show that your score is 10 if you get 8 examples right. What is your score if you get 7 examples right? if you get only 3 examples right?

2. Use the above table to find the scores in these tests:

Number in test	5	5	8	10	12	4	5	16	12	15
Number right	4	5	4	7	11	3	3	13	8	14

KEEPING RECORDS IN IMPROVEMENT TESTS



The best way to keep your record is to make a picture, or *graph*, of your daily scores, as Paul did at the right.

Along the lowest line of the graph Paul wrote the dates on which he tried the tests. He used the abbreviations given below. S 10 means Sept. 10, which is the first day on which Paul tried Test 1 A. The heavy black dot above S 10, on the horizontal line numbered 9, shows that on Sept. 10 his score was 9.

S = Sept.	F = Feb.
O = Oct.	M = Mar.
N = Nov.	A = Apr.
D = Dec.	My = May
J = Jan.	Ju = June

You should have a notebook of squared paper in which to keep your record, as shown above.

Reading the Graph of Paul's Record

1. What score did Paul get on Sept. 12? Did he do as well on Sept. 12 as on Sept. 10? Did he improve again on Sept. 14?

2. How long after he first scored 10 on Test 1 A did he try the test again? Did he do as well on Nov. 12 as on Sept. 14?

3. The first time Paul tried Test 1 B, did he do as well as he did the first time on Test 1 A? What was his lowest score on this test? his highest score?

4. On Test 1 B, on what date did Paul first get a score of 10? How long after that did he wait before trying Test 1 B again?

5. Did Paul do as well on Test 1 C as he did on the other two tests? What was his first score on Test 1 C? his lowest score? his highest score? his last score?

FINDING AND RECORDING SCORES

1. How many examples are there in Test 1 A? Dorothy had 6 examples right in Test 1 A on Sept. 11, 7 examples right on Sept. 12, and 8 right on Sept. 16. Use the table on page 16 to find her score for each of these days.

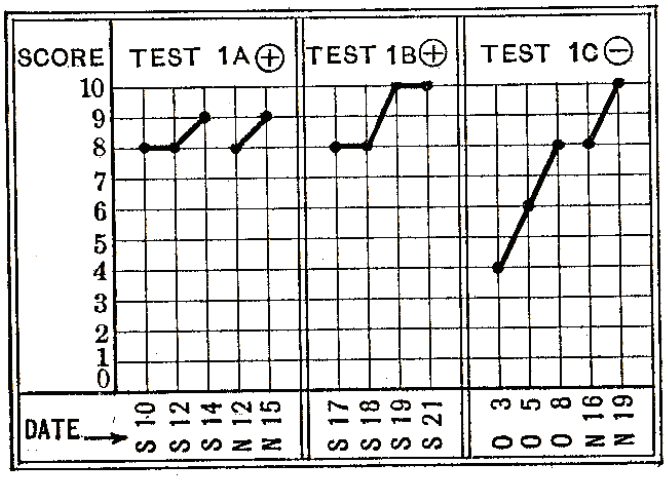
Make a graph to show Dorothy's record on Test 1 A, using the abbreviations given on page 17 for the names of months. Did Dorothy improve steadily on this test?

2. On a test containing 6 examples, Arthur had 5 examples right on Sept. 22, 4 right on Sept. 24, 5 right on Oct. 6, and 6 right on Oct. 8. Use the table on page 16 to find his score on each trial. Make a graph of Arthur's record. Does the graph show a steady improvement?

3. On a test of 12 examples, Leon had 8 examples right on Nov. 4, 9 right on Nov. 5, 9 right on Nov. 10, and 12 right on Nov. 13. On the same test, Claude had 9 examples right on Nov. 4, 7 right on Nov. 5, 8 right on Nov. 10, and 9 right on Nov. 13. Find each boy's daily scores and make a graph of each record.

Whose graph shows the greater improvement, Leon's or Claude's? Which boy made the better record on this test?

4. Here are the graphs of Elsie's records on three tests.

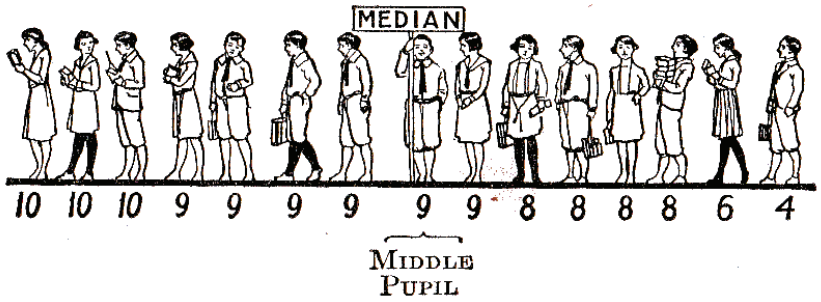


On Sept. 10 Elsie scored 8 on Test 1 A. What was her score each of the other times she tried Test 1 A? Give her score on each day that she tried Test 1 B; Test 1 C.

On which test did Elsie do the best? On which test did she improve the most? On which does she need more practice?

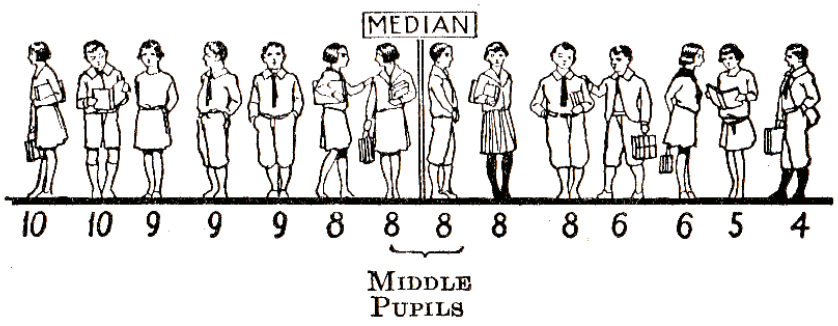
HOW TO FIND THE MEDIAN SCORE

This picture shows the members of a class arranged according to their marks on Improvement Test 1 A. The numbers under them show their scores. Those with the highest scores come first; those with the lowest come last. The middle pupil in the line has the middle score, or the *median score*, of the class.



There are 15 pupils in this class. The middle pupil is the 8th, there being 7 pupils before him and 7 pupils after him. His mark of 9 is the median score of the class for this test. All those who have less than 9 must work very hard to get up to, or ahead of, the median next time.

Here is another class in similar order. When there are 14 pupils, as in this class, there is not one middle pupil; instead, there are 2 middle ones. The 2 middle pupils here both have 8 as their mark; therefore the middle mark of the class is also 8.



Below are the marks of another class of 14 pupils. The two middle pupils have marks of 9 and 8. The median score is

10 10 9 9 9 9 9 ↑ 8 8 8 8 8 6 6
MEDIAN

half way between the two middle scores, or half way between 9 and 8. This gives a median of $8\frac{1}{2}$; but as Improvement Tests are scored by whole numbers, the median is called 9.

WHICH SIDE WILL YOU BE ON?

The median divides a class into two equal groups, representing the upper and lower halves of the class. In which half do you want to be? Will you be before the median flag or behind it? The median is often a high mark; but since at least half of your class has reached it, you also can reach it.

Always try to be among those whose mark is above the median, so that you will be in the upper half of the class.

A Short Way to Find the Class Median

Here is a short way to find the class median:

Consider the first class of 15 pupils, which is pictured on page 19. After the papers are scored, the teacher need not take the time to arrange the pupils in line. She simply asks how many have a score of 10, and 3 pupils raise their hands. She then asks how many have 9, and 6 pupils raise their hands. She writes on the board the number of pupils having each score, as shown above.

Score	*Number of Pupils*	
10	3	
9	6	← Median score in this group
8	4	
6	1	
4	1	
	15	

Since the 8th score is the middle score, the teacher counts down in the column marked "Number of Pupils" until she reaches the 8th pupil. In the first group there are 3 pupils; to these she adds 5 pupils from the next group in order to reach the 8th pupil. Since the 8th pupil belongs to the group of 6 pupils who received a mark of 9, the median score is 9.

NOTE. To find which is the middle score, add 1 to the total number of scores and divide by 2. In the example above, add 1 to 15, which gives 16. Dividing 16 by 2, you get the 8th as the middle score. For a class of 14 pupils, add 1 to 14, which gives 15. Since $15 \div 2 = 7\frac{1}{2}$, the $7\frac{1}{2}$th score, which is half way between the 7th and the 8th score, is the middle score.

FINDING CLASS MEDIANS

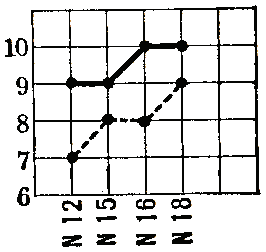
1. In a class, 4 pupils had a score of 10 on an Improvement Test, 7 pupils had 9, 7 had 8, 4 had 7, 2 had 6, and 1 had 3. How many pupils took the test? Arrange the marks in a table, and find the median by the short method given on page 20.

2. On Monday, in Amy's class, 5 pupils had a score of 10 on an Improvement Test, 6 had 9, 9 had 8, 4 had 7, 2 had 6, and 2 had 5. Find the class median.

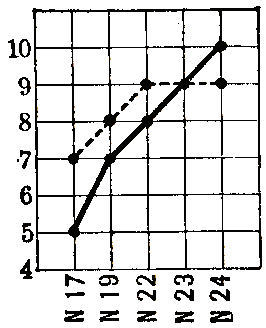
3. On Tuesday Amy's class repeated the test. This time 7 pupils had a score of 10, 7 had 9, 9 had 8, 3 had 7, 1 had 6, and 1 had 5. Find the class median for Tuesday. Had it gone up since Monday?

Comparing Your Work with That of Your Class

In this graph the heavy line shows Anna's record on a test; the dotted line shows the median scores of the entire class on that test. What was the median on Nov. 12? on Nov. 18? What were Anna's scores on those days? Was Anna's work always satisfactory?



The second graph shows Joe's record by a heavy line and that of his class by a dotted line. Joe was far below the class on Nov. 17. On Nov. 19 he did better, but the class median also rose. What was the class score on Nov. 22? What was Joe's score?

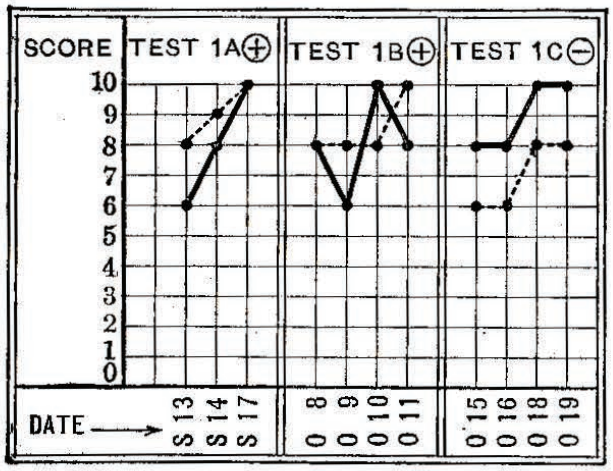


Joe worked hard and continued to improve. How did his score compare with the class median on Nov. 23? on Nov. 24?

For every Improvement Test your class medians will be kept on the board. In your record book, make a graph of the class medians in red and of your own scores in black. Try to keep your line above the class line.

COMPARING RECORDS WITH THE CLASS RECORD

This graph shows Philip's scores (heavy lines) and the medians of his class (dotted lines) on three Improvement Tests.



1. Was Philip above or below the class median when he tried Test 1 A on Sept. 13? Did he improve on Sept. 14? Did the class also improve on Sept. 14? On what day did Philip come up to the class median on Test 1 A?

2. On Test 1 B, how did Philip's score compare with the class median on Oct. 8? on Oct. 9? on Oct. 10? on Oct. 11? Was his work always satisfactory on Test 1 B? Does he need more practice on this test?

3. Compare Philip's score with the class score on each of the days Test 1 C was tried. Did he always make a satisfactory score on Test 1 C? Which needs more practice on this test, Philip or the rest of the class?

4. On Test 1 A, Nora's marks were 8 on Sept. 21, 6 on Sept. 24, 8 on Sept. 26, and 10 on Sept. 28. The medians of her class were 8 on Sept. 21, 8 on Sept. 24, 9 on Sept. 26, and 9 on Sept. 28. Draw a graph of Nora's scores in black and of the class medians in red. On which days was Nora's score below the class median? On which days was her score equal to the class median? above the median?

5. Draw a graph showing Bertha's scores on a test, if her score was 8 on Oct. 5, 10 on Oct. 8, 9 on Oct. 26, and 10 on Oct. 29. On the same graph show, in red, that the class medians on that test were 7 on Oct. 5, 9 on Oct. 8, 8 on Oct. 26, and 10 on Oct. 29. Compare Bertha's graph with the class graph.

SAVING TIME IN ADDITION

In adding a column of figures, you can often save time by grouping the figures. Thus, in adding the columns at the right, look ahead quickly as you go up each column and group mentally the figures whose sum is 10 or less. In adding Column *A*, for example, think 10, 15, 21. In Column *B*, think 7, 17, 23. Always take the figures in their regular order in grouping; thus, in Column *C*, do *not* try to group 5 and 5 or 8 and 2. If you skip around to form a group, you may easily forget the figures you skip.

A	*B*	*C*
6	2	3
3	4	5
2	5	2
4	5	5
6	7	8

In Improvement Tests in addition, you will soon find it much quicker to group the figures than to add one figure at a time. Bookkeepers often group three and four figures at a time.

CHECKING ADDITION

Since an error in adding up a bill or an account may mean the loss of many dollars, the wise business man always checks his computations. Hence it is important for you to form, now, the habit of checking your work.

In an example like that at the right, begin at the bottom of each column and add upwards. *When all the columns have been added, check the work by adding downwards in each column.* You should get the same sum each way.

697
341
258
201
905
217
2619

In an Improvement Test in addition, like Test 1 A, if the teacher wishes the whole class to check the entire test, she will allow twice the given time (8 minutes in all) for doing and checking the work. If the time for this test is only 4 minutes, do not begin to check your work until you have finished the test. Then, if there is any time left before the signal to stop, check as many of your answers as you can.

PRACTICE IN ADDITION

Add the following. Begin at the bottom and add up. Check the work by adding down:

1.	2.	3.	4.	5.	6.
744	605	594	634	719	978
895	666	152	969	794	808
900	636	285	454	100	566
478	270	521	330	122	577
729	471	710	183	689	789
580	860	234	138	390	745
698	823	577	620	134	990
803	179	609	524	306	564
218	437	540	119	532	356

7.	8.	9.	10.	11.	12.
391	947	177	395	505	538
108	882	286	329	717	630
488	446	348	586	413	518
977	738	700	496	676	884
622	267	696	812	347	306
365	846	929	111	725	426
420	950	513	540	540	760
502	153	592	907	625	954
117	439	380	249	456	636

13.	14.	15.	16.	17.	18.
419	787	349	704	227	746
163	217	911	586	906	634
695	586	318	909	275	667
532	838	958	851	794	476
346	658	853	993	445	392
500	765	264	889	152	781
799	210	120	358	260	505
942	669	717	580	404	900
310	570	830	715	980	573
806	404	107	204	667	366
243	321	792	725	259	358

CHECKING SUBTRACTION

The best way to check subtraction is by adding the remainder and the subtrahend; the result should equal the minuend. In checking subtraction, always begin with the remainder and add up in each column.

8052 Minuend
6397 Subtrahend
1655 Remainder

To check the above problem, think as follows:

5 + 7 = 12. The 2 of 12 checks with the 2 of 8052.
1 (carried) + 5 + 9 = 15. The 5 of 15 checks with the 5 of 8052.
1 (carried) + 6 + 3 = 10. The 0 of 10 checks with the 0 of 8052.
1 (carried) + 1 + 6 = 8. The 8 checks with the 8 of 8052.

As soon as you finish working an Improvement Test in subtraction, begin at once to check it, checking as many of the problems as you can in the time allowed for the test. If possible, the teacher will sometimes give you twice as long as the stated time allowance, so that the entire class can work and check all the problems.

Exercises

Subtract and check the following:

1.	5171 2090	**3.**	7260 5459	**5.**	6485 3892	**7.**	2843 1756	**9.**	3111 2999
2.	8643 1916	**4.**	9276 8579	**6.**	9065 438	**8.**	6422 2466	**10.**	8343 2965

Work the following problems and check your results:

11. Find the difference between \$18.29 and \$76.30.

12. How much more than \$61.05 is \$63.01?

13. When Mrs. Bates went shopping she had \$18.70. When she returned home she had \$9.85 left. How much did she spend?

14. If you have \$4.36 and need \$5.25 to buy a tennis racket, how much less have you than you need?

PRACTICE IN SUBTRACTION

Subtract the following and check the work:

1. 4603 2469	**2.** 7522 4751	**3.** 8341 6802	**4.** 6000 1834	**5.** 13751 4898
6. 9637 7952	**7.** 3568 1039	**8.** 4000 2055	**9.** 7156 3887	**10.** 35417 6893
11. 9062 2300	**12.** 7143 5164	**13.** 8652 4653	**14.** 8206 1496	**15.** 48975 8597
16. 8721 3713	**17.** 8000 5742	**18.** 7629 1892	**19.** 9424 4760	**20.** 31097 7519
21. 5030 3177	**22.** 9281 6687	**23.** 6417 2400	**24.** 6150 2516	**25.** 20345 8265

Subtract the following and check the work:

26. \$741.98 477.69	**27.** \$150.00 35.23	**28.** \$3363.68 1566.00	**29.** \$6449.26 101.37
30. \$605.03 441.79	**31.** \$590.52 81.48	**32.** \$4641.34 2884.27	**33.** \$7000.00 526.43
34. \$800.00 661.39	**35.** \$800.52 47.96	**36.** \$6341.38 3342.32	**37.** \$8868.28 887.55
38. \$531.93 251.93	**39.** \$176.52 97.54	**40.** \$6000.00 1524.86	**41.** \$1416.81 986.29
42. \$603.51 269.86	**43.** \$600.00 18.32	**44.** \$6385.38 5799.07	**45.** \$4203.78 284.37

HOW CHANGE IS MADE

Albert buys a notebook for 13¢ and gives the clerk $1.00. The clerk makes the change very quickly.

Albert thinks that the clerk first subtracts 13¢ from $1.00, which leaves 87¢, and then picks up 87¢ in coins.

Jane says that the clerk does not subtract at all, but makes the change by adding, thus: He thinks 13¢, then he picks up 2 cents and thinks 15, then he takes a dime and thinks 25, next a quarter and thinks 50, and then a half dollar and thinks $1.00. As he puts the change into Albert's hand he counts again, saying, "13, 15, 25, 50, $1."

Jane is right. In making change the clerk does not subtract; he starts with the amount of your purchase and adds coins to that amount until he reaches a dollar. Then he hands you the coins. When you receive the change, you should check it by adding in the same way.

Exercises

1. Marie buys a book for $1.29 and gives the clerk $2. He gives her 1 cent, 2 dimes, and a half dollar. Count the change as the clerk does. Is it right?

2. If Thomas bought a battery from you for 58¢ and gave you $1, what coins would you give him as change? How would you count them out?

Tell what coins you would give as change from $1 *on each of these purchases, and count out the change:*

3. 45¢ **4.** 25¢ **5.** 28¢ **6.** 83¢ **7.** 69¢ **8.** 92¢

Make change from $5 *for each purchase and count it out:*

9.	$2.50	**11.**	$3.70	**13.**	$4.12	**15.**	$1.80	**17.**	$3.39
10.	$2.08	**12.**	$1.05	**14.**	$0.46	**16.**	$1.99	**18.**	$4.35

TWO-STEP PROBLEMS

1. Alice bought a sweater for $2.89 and a skirt for $2.25. If she gave the clerk a 10-dollar bill to pay for these things, how much change did she get?

2. Mr. Case spent $1.75 for one book and $2.35 for another book. He gave the clerk $5 in payment for them. How much change did Mr. Case get?

3. George had $15.00. He bought 14 hens at $.85 each. How much money did he have left?

4. Henry had a collection of 450 coins of all kinds. He sold 75 coins to Frank and gave 33 coins to his brother. How many coins did Henry have left?

5. In the summer James works 7 hr. a day in Mr. Foster's garden. If he is paid 25¢ an hour, how much in all does he earn in 15 da.?

6. Bert saved $33.85 of the money he earned last summer. His father gave him $5 for his birthday. If he wants to buy a radio set costing $45.50, how much more must he save?

7. At the store Mrs. Adams paid $.83 for fresh vegetables, $.45 for sugar, $.35 for coffee, and $.38 for eggs. How much change did she get from $5.00?

8. Mary has been saving $.45 a week for 28 wk. She wants to buy a bicycle that costs $19.75. Her father says he will give Mary the rest of the money. How much will her father pay?

9. Ann practiced on the piano 45 min. on Monday, 50 min. on Tuesday, 40 min. on Wednesday, 45 min. on Thursday, 30 min. on Friday, and 60 min. on Saturday. Find the average number of minutes she practiced daily.

10. Mr. West has an apple orchard. He gave his son, Henry, 24 bu. of apples to sell. Henry sold all of them at $1.75 a bushel. He paid his father $25.00 for the apples. How much did Henry make by selling the apples?

HOW TO CHECK MULTIPLICATION

Fred has multiplied 273 by 812. To find out if his answer is correct, he may check it by any of these methods:

1. *By doing the work over.* If he does the example again and gets the same result, his work is probably correct. This is the best check when the multiplier has only one figure.

2. *By reversing the factors.* He may exchange the positions of the numbers and multiply again. For example, after multiplying 273 by 812, he can check the work by multiplying 812 by 273. The product should be the same either way.

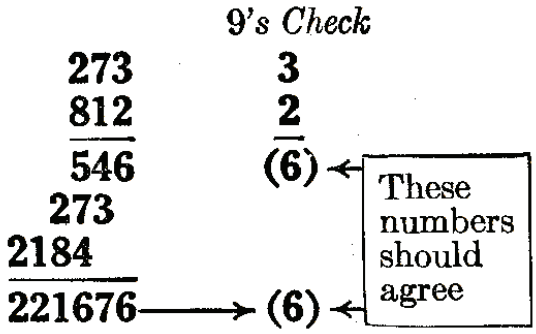
3. *By casting out 9's.* This method, discovered by the Arabs many centuries ago, is the shortest check for multiplication when the multiplier contains two or more figures.

Checking Multiplication by Casting Out 9's

You check multiplication by "casting out 9's" like this:

First add the digits of 273, thus: $2 + 7 + 3 = 12$. There is one 9 in 12. If you subtract, or "cast out," this 9, you have 3 left. This 3 is the "check number" of 273.

Find the check number of 812 thus: $8 + 1 + 2 = 11$. $11 - 9 = 2$. The check number of 812 is 2.



Now multiply the first check number, 3, by the second, 2. This gives 6 as the final check number. Write 6 in parentheses.

Finally, find the check number of the product, 221,676, thus: $2 + 2 + 1 + 6 + 7 + 6 = 24$. Casting out both of the 9's in 24, you have 6 as the check number. Since this is the same as the final check number, 6, obtained above, the answer is correct.

Multiply the following and check by casting out 9's:

1.	**2.**	**3.**	**4.**	**5.**
346	813	2422	1805	2612
119	344	71	667	993

A QUICKER WAY TO FIND CHECK NUMBERS

In the example at the right, the sum of the digits of 296 is 2 + 9 + 6, or 17. Instead of subtracting 9 from 17 to get 8 as the check number, you can find 8 more quickly by adding the digits of 17, thus: 1 + 7 = 8. Similarly, the sum of the digits of 626 is 14, and 1 + 4 = 5.

296	**8**	
626	**5**	
1776	**40**	**(4)**
592		
1776		
185296		**(4)**

Multiplying the check number 8 by the check number 5, you get 40. Adding the digits of 40, you get 4 as the final check number, which must always be less than 9.

Next add the digits of 185,296, which gives 31. Adding 3 + 1, you get 4 as the check number. Since this 4 agrees with the final check number, 4, the work is considered correct.

To find a check number by adding the digits, you may skip a 9 or any digits whose sum is 9. In finding the check number of 185,296, you need add only 5, 2, and 6. The check number of 7245 and of 4320 is 0.

Study these examples, which are checked by casting out 9's:

429	**6**	
174	**3**	
1716	**18**	**(0)**
3003		
429		
74646		**(0)**

This work is considered correct because the final check numbers agree.

554	**5**	
168	**6**	
4432	**30**	**(3)**
3224		
554		
92072		**(2)**

This work is incorrect because the final check numbers do *not* agree.

Exercises

Multiply the following and check by casting out 9's:

1.	**2.**	**3.**	**4.**	**5.**
3298	793	2409	3298	4577
364	512	673	819	2736

6. In multiplying 888 by 666, Frank got 491,408, Harry got 393,407, and Arthur got 591,408. Who was right?

WATCHING THE PARTIAL PRODUCTS

1. Robert checked this example by casting out 9's. His final check numbers were both 5; but when he came to school, he found that his answer was not correct. Can you tell what is wrong?

557	8	
67	4	
3899	32	(5)
3342		
338099		(5)

Robert wrote the partial product 3342 one place too far to the left. Even though this example checks by 9's, the work is incorrect.

2. In doing this example, Mary put the partial product 876 one place too far to the right. Does her example check by 9's in spite of the error?

438	6	
205	7	
2190	42	(6)
876		
10950		(6)

Since the check of 9's does not show when a partial product has been misplaced, be sure that each partial product is in the right place before you cast out 9's.

It also rarely happens that two errors in a problem counteract each other so that the check of 9's does not disclose the errors. These exceptions are of minor importance, however, since the check of 9's detects 95% of all errors that occur and is by far the quickest check for multiplication and division. It is not recommended for addition and subtraction.

Exercises

Multiply. Check each answer by casting out 9's:

1.	724	**2.**	829	**3.**	457	**4.**	466	**5.**	715	**6.**	235
	386		297		928		639		452		869

If the partial products are in the right place, check by casting out 9's. Work again and check the incorrect examples:

7.	843	**8.**	265	**9.**	764	**10.**	682
	97		18		309		501
	5901		2120		6776		682
	7587		265		2292		3410
	81771		28620		29696		341682

PRACTICE IN SUBTRACTING AND MULTIPLYING

Subtract the following and check the work:

1. \$982.18 398.52	**2.** 72561 6983	**3.** 385886 338969	**4.** 314455 28977
5. \$500.00 145.89	**6.** 62006 2784	**7.** 737597 288668	**8.** 900090 36690
9. \$627.43 291.67	**10.** 49000 8125	**11.** 638266 154497	**12.** 425324 96673
13. \$523.47 275.52	**14.** 40000 2894	**15.** 941132 141734	**16.** 783488 71445

Multiply the following and check the work:

17. 27 × \$78.09	**31.** 204 × 249	**45.** 809 × 7005
18. 39 × \$14.59	**32.** 560 × 214	**46.** 618 × 4324
19. 57 × \$56.34	**33.** 135 × 390	**47.** 427 × 4570
20. 86 × \$80.90	**34.** 708 × 612	**48.** 670 × 5030
21. 28 × \$70.05	**35.** 942 × 316	**49.** 405 × 8159
22. 46 × \$53.67	**36.** 109 × 902	**50.** 230 × 3456
23. 98 × \$36.04	**37.** 953 × 782	**51.** 437 × 1002
24. 53 × \$60.78	**38.** 206 × 760	**52.** 816 × 8745
25. 49 × \$70.08	**39.** 678 × 809	**53.** 721 × 1028
26. 35 × \$25.09	**40.** 406 × 471	**54.** 865 × 4376
27. 83 × \$36.85	**41.** 128 × 850	**55.** 175 × 2918
28. 19 × \$42.50	**42.** 294 × 902	**56.** 326 × 7005
29. 42 × \$18.34	**43.** 304 × 760	**57.** 263 × 2840
30. 51 × \$54.86	**44.** 150 × 918	**58.** 449 × 1789

IMPROVEMENT TEST No. 2

Test 2 A — Multiplication (Time 4 min. after copying)

1. 62×351	**2.** 73×173	**3.** 19×587
4. 84×902	**5.** 26×694	**6.** 26×278
7. 59×326	**8.** 38×173	**9.** 85×894

Test 2 B — Multiplication (Time 4 min. after copying)

1. 769×253	**2.** 947×847	**3.** 533×813
4. 918×964	**5.** 562×657	**6.** 475×929

Test 2 C — Multiplication (Time 3 min. after copying)

1. 957×8146	**2.** 508×3527
3. 610×9057	**4.** 324×4081

Test 2 D — Division (Time 4 min. after copying)

Express remainders, if any, as fractions in the quotient:

1. $78\overline{)24736}$	**2.** $92\overline{)42059}$	**3.** $61\overline{)43249}$
4. $47\overline{)10293}$	**5.** $38\overline{)32137}$	**6.** $85\overline{)30435}$

Test 2 E — Division (Time 4 min. after copying)

Express remainders, if any, as fractions in the quotient:

1. $64\overline{)41440}$	**2.** $56\overline{)50579}$	**3.** $71\overline{)37133}$
4. $31\overline{)34435}$	**5.** $39\overline{)29172}$	**6.** $58\overline{)33263}$

NOTE. In an Improvement Test in multiplication or division, first copy all the examples. The time allowance does not include the time required for copying. If 1 min. extra is allowed on each test, all the examples can be checked by casting out 9's, as shown on pages 30 and 35.